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Productive Connective Thinking Scheme in Mathematical Problem Solving

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ABSTRACT

The need to make connections in doing problem-solving has been the focus of many researchers. The purpose of this study was to describe the formation of productive connective thinking schemes when students do complete the phases of mathematical problem-solving. Schemes are formed through the phases of cognition, inference, formulation, and reconstruction. This qualitative study adopted several modes of data collection to triangulate data to ensure the validity of the findings. Twenty four research participants were selected among high performing Grade 12 students of three Indonesian Secondary Schools using the purposive sampling method. Productive thinking schemes were identified based on the analysis of the participants' written assignment, think-aloud recordings and interview transcriptions. Description of the schemes was concluded from the understanding of the causes of the problems and the way the participants make associations between ideas. Students' thinking structure is aligned with the structure of the given problem. In solving the problem, participants formed constructive thinking schemes which were generalization schemes that require high spatial and abstraction abilities. This allows reconstruction of the connective thinking network scheme that forms a new connective scheme that can be used for more complex problem-solving.

Keywords: Connective thinking schemes, mathematical problem solving, schemes, thinking

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INTRODUCTION

Various research has been carried out by researchers to improve the ability to solve the mathematical problem (Intaroset al., 2014; Kapur, 2010; Krawec, 2014; Rosli et al., 2013; Schoenfeld, 2013, 2016). However, student learning outcomes on mathematical problem solving are still

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below the average standard, as shown by the PISA 2015 research data on mathematical literacy skills of students from 72 countries (Organisation for Economic Co-operation and Development [OECD], 2017).

Mathematical problem solving requires students to think mathematically which involves the use of mathematical concepts, procedures, facts and principles (OECD, 2017; Saad & Ghani, 2008). Additionally, students must have the ability to describe, explain and predict phenomena (Intaros et al., 2014). Research findings on students' success in solving mathematical problems show that students must have high creativity, metacognitive skills, analogy skills, ability to connect among concepts, other sciences, and everyday life, and ability to use manipulation strategies (Eli et al., 2013; Knox, 2017; Krawec, 2014; Matejko & Ansari, 2015; Schoenfeld, 2016; Susanti et al., 2013; Turmudi & Susanti, 2018).

National Council of Teachers of Mathematics (NCTM) posited that there was a relationship between mathematical problem-solving and mathematical connections (NCTM, 2015). This was also emphasized by Mhlolo et al. (2012) who stated that students' ability to connect between mathematical concepts was an important factor in aiding students to solve mathematical problems. According to Faidah and Susanti (2017) as well as Tasni et al. (2017), at this stage, these students have already acquired productive thinking skills. Furthermore, students who have this ability have a tendency to construct their ideas in connecting mathematical concepts and then generalizing it to more complex mathematical problems (Fuchs et al., 2004).

Research on thinking ability is profoundly rooted in cognitive theory, proportional thinking, analogical thinking, reflective thinking, creative thinking, critical thinking (Aloqaili, 2012; Dubinsky, 2002; Jitendra et al., 2011; Oxman, 2017). However, none of the studies had focused on the flow of thoughts of students during the problem-solving process. In cognitive theory, the scheme of thinking is related to the description of the arrangement and sequence of ideas built by someone in the structure of their thought (Chalmers, 2003; Schmidt, 2003).

Based on this background, the researchers were interested to examine how students who are high performers in mathematics build the network formation scheme to enable them to construct concepts and productively connect concepts in solving mathematical problem-solving. The purpose of this study was to describe the formation of productive connective thinking schemes when these students undergo the phases of mathematical problem-solving. Representation is one way for students to build ideas when connecting mathematical concepts. Thus, students' cognitive activity in establishing mathematical connections in solving mathematical problems was explored by analyzing the representations made by the students.

MATERIALS AND METHODS

Mathematical Connections

Mathematical connections refer to the cognitive process in connecting or associating two or more ideas, concepts, definitions, theorems, procedures, and representations in math, with other disciplines, including real-life (Garcia-Garcia & Dolores-Flores, 2018). In the Principles and Standards for School Mathematics (NCTM, 2000), problem-solving and making connections are stated as part of the five process standards. NCTM (2015) had also highlighted the significant correlation between problem-solving and mathematical connections. In the process of problem-solving, McLeod (2018) explained that the understanding of what information is known and what is asked about the problem becomes the initial factor in understanding the cause of the problem. Furthermore, part of the solution is needed to build ideas in connecting mathematical concepts (Faidah & Susanti, 2017; Susanti, 2015; Turmudi & Susanti, 2018).

In order to understand the causes of mathematical problems, students must first make mathematical connections (Tambychik & Meerah, 2010). They must be able to connect the information given with what the problems want. Problem-solving requires a connection between mathematical concepts, between concepts and other sciences and between concepts and everyday problems. This connection process can be described as a spider network where dots are mathematical concepts and threads that connect between points as connections (Tall & Barnard, 1997). This spider network describes the cognitive activity which is the relationship between the cause and solution of the problem.

Cognitive activities evolved in the construction of mathematical connections can be analyzed from the process of representation carried out when building reciprocal relationships between mathematical concepts (Eli et al., 2013; Susanti, 2015), may it be concrete or abstract representation. The relationship of concrete representations and abstract representations is that cognitive activities can be represented simultaneously (Uttal et al., 2009).

Productive Connective Thinking

Connective thinking takes place when the learner makes a connection between mathematical ideas. As elaborated by Slavin (2008), the connective thinking process takes place in the working memory, involving the formation of connections between new and old knowledge that have the same and interrelated meaning to form a connective thinking scheme.

In this study, representation becomes a tool to describe the process of thinking of the subject during the process of communicating ideas to build connections. Representation is a model or a substitute form of a problem situation or aspect of a problem situation that is used to find a solution. For example, problems can be represented by objects, images, words, or mathematical symbols (Ellis, 2007; Lesh et al., 1987; Otting et al., 2010; Tall & Barnard, 1997). Hence, mathematical representation communication of mathematical ideas from the thought process, either using language, diagrams, graphics, symbols, tables, spatial, numerical and verbal, so that they become facts that can convince others. as convincing facts. Students who are able to build connections in solving mathematical problems are regarded as having the tendency to think of productive connections. Thus, the activities in this cognitive process, include (i) the active role of students in building meaningful knowledge, (ii) the importance of making connections between ideas in construction significantly, and (iii) the importance of connecting between ideas and new information.

Cognitive Scheme Formation Theory

According to Toshio (2000), schemes are general units and standards of mental structure used systematically at all times to make decisions or take behavior. Similarly, it is also used by von Glaserfeld and proponents of Constructivism. Through one's cognitive process new knowledge is constructed based on experience (Root et al., 2017) and new knowledge gained will be stored in one's long-term memory in the form of a scheme. The theory of cognitive scheme formation by Toshio (2000) was used to explore ideas in the formation of mathematical connections. Based on Toshio (2000), the phases of forming cognitive scheme are as follows:

(i) Cognition phase: understanding the problem situation and thinking about the direction of problem-solving;

(ii) Inference phase: find appropriate information and a reasonable and logical basis for planning problem solving;

(iii) Formulation phase: verifying problems, deciding to process and discovering new knowledge through mathematical schemes; and

(iv) Reconstruction phase: looking back, evaluating, reconstructing the entire problem-solving process then generalizing ideas to other domains.

Research Methods

This qualitative descriptive study had adapted several modes of data collection. In ensuring the validity of findings, data needs to be triangulated from several resources. The different source used for data collection was aimed to illustrate the scheme of students' productive connective thinking in building mathematical connections when solving mathematical problems.

The participants of this study were 24 high performing students who were in Grade 12 from three Secondary Schools in East Java Indonesia. High performing students were chosen because they were able to go through all the stages of problem-solving. To select the research participants, 120 students were first given the initial assignment on problems that required making connections between concepts. The written test as an instrument of study covers problem-solving items related to functions, number patterns, sequence patterns, the general formula of the n^{th} term of a sequence of numbers.

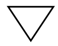
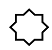
The task is in the form of multilevel stupas which are composed of several cube units. Each level forms a number pattern that can be generalized into a general formula. This instrument was validated by three experts mathematics education, pure mathematics, and psychology. Scoring with a scale of 0-100 was imposed on students' answers based on connection indicators and the results are grouped into three categories of high (85-100), moderate (60-84) and low (0-59) student abilities. This was used to determine their abilities. Based on the assessment, 24 students were categorized as high performers, 45 as moderate and 51 as low performers. Hence, the 24 students in the high performing group were chosen as research participants.

The selection of research participants had adopted the purposive sampling method because it is considered as the most effective method for studying certain domain experts who can provide the information needed (Tongco, 2007). The selection of subjects was based on three considerations as follows: (i) The three schools selected represents the city and district; (ii) The participants were able to provide rich source of information on network connection thinking scheme in mathematical problem-solving.; and (iii) 12th grade students were chosen because they already had all the basic concepts related to problem-solving.

Data was collected from the assignment that requires making connections in solving the given problems through the thinking aloud procedures. Their activities were noted and recorded while they were solving the problems and was later followed by in-depth interviews with the participants who have met the criteria. This allowed the researchers to complete the data obtained from the think-aloud procedures. The data collection was carried out until saturation occurred, which meant that the same data characteristics were obtained. In other words, it was done until no new information evolved from the interviews. Finally, only 12 participants were selected as informants in the triangulation process for data validity. Productive thinking schemes were identified based on the analyses of the participants' written assignments, think-aloud recordings, and interviews. The data obtained in the form of student work from the written tests that describe the thought process. Think aloud and interview strategy reinforces the explanation of what participants think when solving mathematical problems. Data from think-aloud obtained through the pronunciation of something that participants thought related to problems. The results of the data collecting activities are then transcribed and coded. The codification is based on the students' ideas that come up while establishing connections when solving a mathematics problem.

Terms and symbols of the occurrence of connective thinking in solving mathematical problems are coded as Table 1. For example, in a stupa problem situation, students determine the stupa image compiler in the form of a unit cube (I1) and in identifying what was known from the stupa drawings, students' responses determine the location of the cube of each storied drawing (I2).

Table 1
Codification of the connection thinking unit

Term	Coding	Symbol	Term	Coding	Symbol
Problem information	Info *p*, *p = 1,2,..,m*		Solution to problems	Sm*x*, *x = 1,2,..,q*	
Ideas that are relevant to the information provided	I*i*, *i = 1,2,3,...*		Productive connection	Kp*b*, *b = 1,2,...,d*	
Relevant ideas with problems given	I*i*, *i = 1,2,..,n*		Change problems	Km*r*, *r = 1,2,3,...,v*	
Ideas, not relevant to problems	It*j*, *j = 1,2,...,l*		Symbolic representation	Rep. Sb	
Relevant ideas outside the problem	Il*s*, *s = 1,2,..,t*		Spatial representation	Rep. Sp	
A relevant idea in reaching a solution to a problem	Isg, *g = 1,2,3,..,h*		Numeric representation	Rep. Num	
Verbal representation	Rep.verb		Representation of pictures	Rep. Gb	

RESULTS AND DISCUSSION

The discussion on the findings of the research is organized based on the objective of the study, which was to describe the formation of productive connective thinking schemes when these students undergo the phases of mathematical problem-solving. Schema formation as advocated by Toshio (2000) covered four phases, cognition, inference, formulation, and reconstruction. Therefore, analysis of data uncovers students' thought processes when establishing mathematical connections scheme from ideas relevant to the problem through four phases of schema formation. The written work data and the thought process through think aloud form the basis of the analysis. The thought process is described through thinking schemes in the form of spider webs. Analysis of data was carried out on the results of the triangulation of students who had a tendency to think productively in processing information on the part of the problem and its solution (Aloqaili, 2012; Jitendra et al., 2011; Oxman, 2017; Xin, 2008).

Cognition Phase

At the cognition phase, students build an understanding of the relationship of problem situations and intend to explore the direction of problem-solving, the information known and the question asked. To understand information about what is known, the participants read the three information provided about the problems. The problem is a non-routine task that requires a connection between concepts and everyday life, in the form of multilevel stupas which are composed of several cube units. the three information in each question is symbolized by a, b and c. Information a is a description of the picture and problem situation, information b is the number of numbers in the pattern to the one from the known image, and information c is the number of numbers in the second pattern.

The structure of students' thinking forms a network of connections between ideas built on the information provided from previously constructed ideas (Anderson et al., 2014). The network connection of the students' thought processes is explained in the connecting thinking scheme in Figure 1.

In Figure 1, based on the information structure of the problem given, idea I1 appears from information a. Students establish connections (KSP1) between information and ideas I1 to bring up ideas I2 which was done through spatial representations (Rep. Sp1). Connection (KSP2) between idea I2 and information b raised a new idea I3 through image representation (Rep.Gb1). Based on KSP2, students built a relationship between idea I3 and information c through verbal representation (Rep.Verb1). Connection (KSP3) between idea I3 and information c raises a new idea of idea I4 is explained in Table 2.

Based on the flow of connective thinking related to the cognition phase, the structure of students' thinking according to the structure of the problem given (Baum et al., 2005; Proulx et al., 2005). The thought structure of the students allowed them to recognize the structure of the problem well. Hence, students did not have difficulty understanding the problem situation and thinking about the direction of problem-solving. The idea that emerges based on the representation constructed showed that the structure of students' connective thinking was in line with the structure of the problem (Barrouillet, 2015; Piaget, 1964, 1983). Furthermore, new ideas were built on the connection between the two previous ideas.

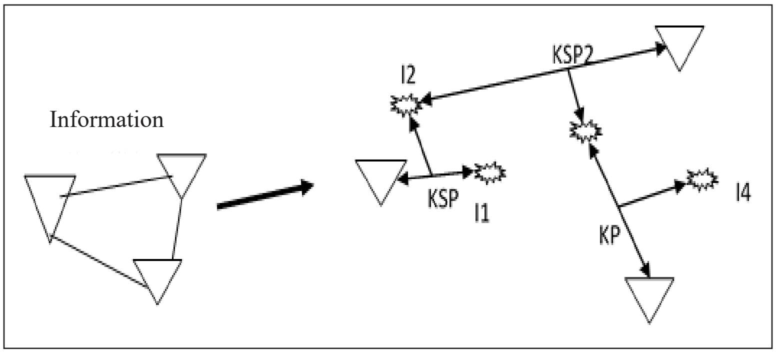


Figure 1. Scheme of productive thinking connection at the cognition phase

Table 2
Connection flow with the representation of cognition phase

Connective Thinking Flow	Aspects of Thinking Cognition Phase
KSP1=a⇔I1⇒I2, KSP2=I2⇔b⇒I3, KSP3=I3 ⇔*c*⇒I4	KSP1 (Rep.Sp1), KSP2 (Rep.Gb1), KSP3 (Rep. Verb1).

Inference Phase

In the inference phase in Figure 2, students find suitable information for the solving of problems and make the inference. Students look for the right information and a reasonable and logical basis for planning problem-solving. In building relationships between representations, students drew and made tables as the first step in planning problem-solving. This is done to complement the understanding of the problems that have been formed at the cognition phase (Tasni & Susanti, 2017).

Participants used the development of reasoning, which was formed cognitively to help them carry out the conclusion phase. Based on the description and analysis of the data above, the researchers describe the thinking process of students in the scheme of thought network connection structure as Table 3.

The connection between ideas is built from the relationship between the pieces of information stored in one's cognition in changing information a, b and c (Courchesne et al., 2011; Solso, 2003). Some relationships

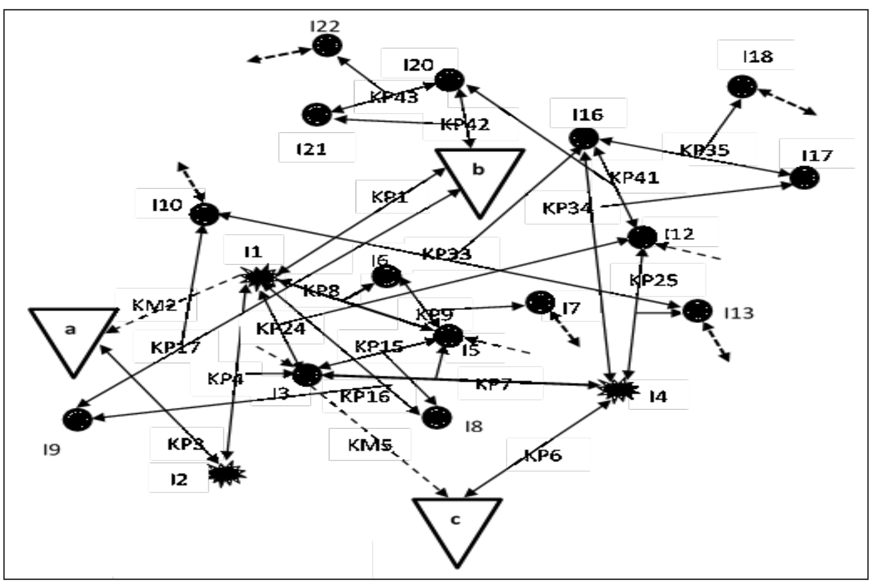


Figure 2. Connected thinking of network scheme at the inference phase

Table 3
Connection flow with the representation of the inference phase

Connective Thinking Flow	Aspects of Thinking Inference Phase
KP8=I5<->I1->I6, KP9=I6<->I5->I7, KM14=SM1<-> I5 KP15=I5<->I3->I8, KP16=II8<->I1->I9,KP17=I9 <->b->I10, KM23=SM2<->I3, KP24=I3<->I1->I12, KP25=I12<->I4->I13,KM32=SM3->I10, KP33=II10<-> 113->I16,KP34=II6<->I4->I7, KP35=I16<->I17->I8, KM41=SM3<->I12, KP42=I12<->I16->I20,KP43=I20<->b ->I21, KP44=I21<->I20->I22.	KP8(Rep.Num1), KP9(Rep.Verb3), KP115(Rep.Sp3), KP116(Rep.Gb3), KP17(Rep.Sp4), KP24(Rep.Sp7), KP25(Rep.Sp8), KP33(Rep.Sp11), KP34(Rep.Sp12), KP35(Rep.Sp13), KP42(Rep.Sp14), KP43(Rep.Sp15), KP44(Rep.Sp16).

between ideas are related and have the same meaning are connected to each other to form new networks as schemes of thought networks that connect like spiders' nests (Hiebert, 1999).

Formulation Phase

Students verify the problem and acquire relevant knowledge and schema**.** The formulation phase is the thinking phase to explore connecting ideas and problems with the process of thinking that has been formed at the phase of cognition and inference phases. A network connective thinking scheme is the concept of knowledge that is formed in the structure of students' thinking based on experience. The concept and scheme of knowledge are basically the same. If one looks in detail from the concept into the knowledge scheme and can capture the scheme as a whole of the connection process then the scheme can be considered a global concept (Baum et al., 2005; Proulx et al., 2005; Zimmer, 2008).

The scheme of student productive connective thinking networks is a complex network of connections (Ashby, 1957; Sternberg, 2013). Although not many connection networks are formed, the ideas that emerge in the scheme of collective thinking are varied and complete as explained in Figure 3.

Figure 3 shows ideas that arise in building connections are ideas based on information, previous ideas, solutions to problems, experience, and generalizations. On each network, old ideas work with other interrelated ideas, then be interpreted in various possibilities with several networks that form structures with many reciprocal relationships. (Barrouillet, 2015; Huang et al., 2012; Nuessel, 1986; Proulx et al.,

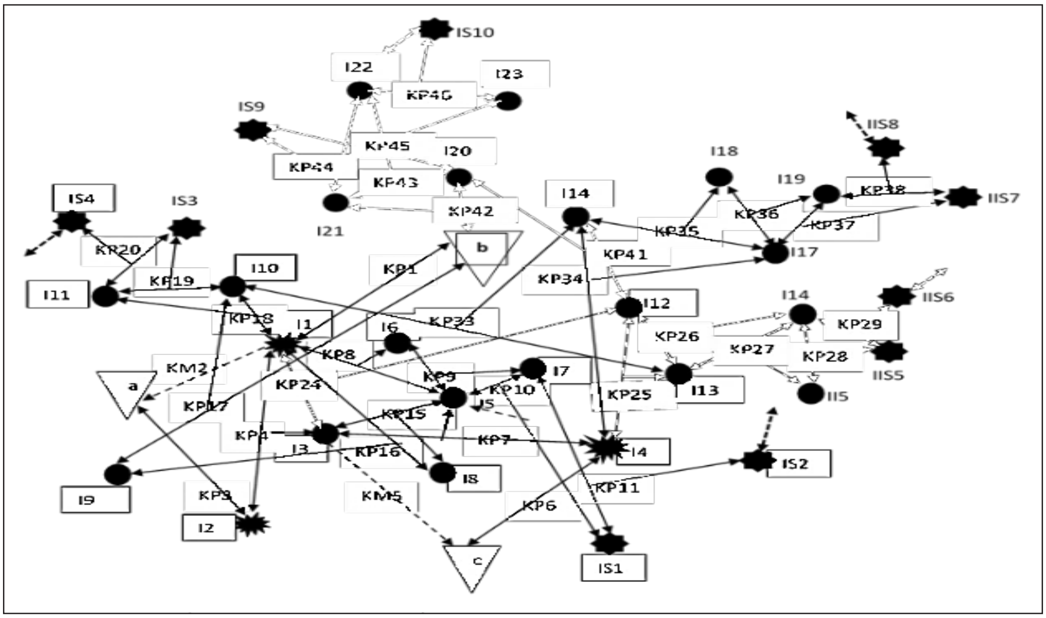


Figure 3. Scheme of productive thinking connected thinking network at the formulation phase

2005; Root et al., 2017). Explanation in the following Table 4.

Reconstruction Phase

In the previous phase, the structure of students' thinking has been formed according to the structure of the problem given. The network scheme considers the connection structure that is formed by the formulation stage which then formed a complex connection. In the reconstruction phase in Figure 4, connective thinking networks that have the same relevance and meaning form relationships through numerical representation, spatial images, and algebra. This representation shows that the student recognizes the relationship between procedures to solve problems one to another procedure and can make connections between mathematical concepts and everyday life (NCTM, 2015; Wienert & Helmke, 2008).

Table 4
Connection flow with the representation of the formulation phase

Connective Thinking Flow	Aspects of Thinking Formulation Phase
KP10=I7⇔I5→I8, KP11=I8⇔I7→I9, KP18=I10 ⇔I1→I11,KP19=I11⇔I10→IS3, KP20=IS3⇔ I11→IS4,KP26=I13⇔I12→I13, KP27=I14⇔I13 →I15,KP28=I15⇔I14→I1S5, KP29=IS5⇔I14 →IS6, KP36=I18⇔I17→I19, KP37=I19⇔I17→ IS7, KP38=IS7⇔I19→IS8, KP45=I21⇔I22→IS9, KP46=IS9⇔I20→I23, KP47=I23⇔I22→IS10	KP10 (Rep.Num2), P11 (Rep.Num3), KP18 (Rep.Sp5), KP19 (Rep.Sp6), KP20 (Rep. Num4), KP26 (Rep.Sp9), KP27 (Rep.Sp10), KP28 (Rep.Num7), KP29 (Rep.Num8), KP36 (Rep.Num9), KP37 (Rep.Num10), KP38 (Rep.Num11), KP45 (Rep.Num13), KP46 (Rep.Sp17), KP47 (Rep.Num14).

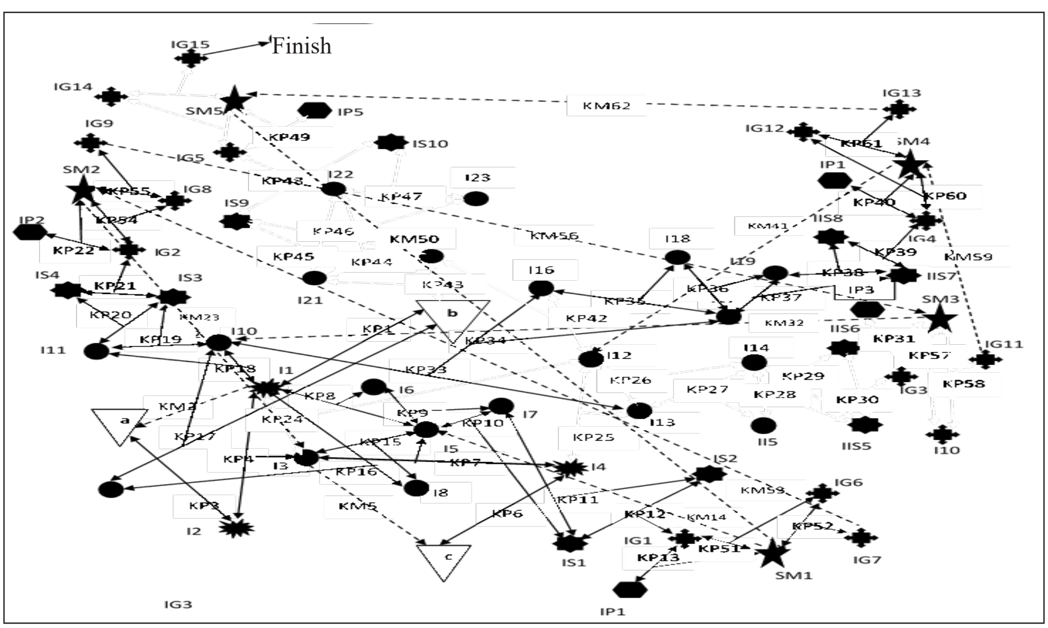


Figure 4. Connected thinking skeptic network scheme at the reconstruction phase

In this process, students get ideas based on their experience so that they can relate understanding and related mathematical concepts (Catrambone & Holyoak, 1989; Doumas & Hummel, 2005; Gick & Holyoak, 1980, 1983). The relationship between ideas is thoroughly reconstructed to form a connective thinking network scheme (Hurlbert, 1986; Matejko & Ansari, 2015; Mousley, 2004; Pugalee, 2001). Thus, the scheme of connective thinking formed is a knowledge network that forms a hierarchy and is structured. As a result, the scheme of connecting thinking can be generalized to solve more complex questions. Explanation in the following Table 5.

Table 5
Connection flow with the representation of the reconstruction phase

Connective Thinking Flow	Aspects of Thinking Reconstruction Phase
KP12=IS1<->IS2->IG1,KP13=IG1<->IP1->SM1,KP21=IS3 <->IS4->IG2,KP22=IG2<->IP2->SM2,KP30=I14<-> IS5->IG3,KP31=IG3<->IP3->SM3,KP39=IS7<->IS8 ->IG4,KP40=IG4<->IP4->SM4,KP47=IS9<->IS10-> IG5,KP48=IG5<->IP2->SM5,KM50=SM5<->SM1,KP51=SM1 <->IG1->IG6,KP52=IG6<->SM1->IG7,KM53=IG7<-> SM2,KP54=SM2<->IG2->IG8,KP55=IG8<->SM2-> IG9,KM56=IG9<->SM3,KP57=SM3<->IG3->IG10,KP58=IG10 <->SM3->IG11,KM59=IG11<->SM4,KP60=SM4<->IG4 ->IG12,KP61=IG12<->SM42->IG13,KM62=IG13<-> SM5,KP63=SM5<->IG5->IG14,KP64=IG14<->SM5->IG15.	KP12(Rep.Num5), KP13(Rep.Alj1), KP21(Rep.Num6), KP22(Alj2), KP30(Rep.Num), KP31(Rep.Alj3), KP39(Rep.Num12), KP40(Rep.Alj4), KP47(Rep.Num15), KP48(Rep.Alj5), KP51(Rep.Num16), KP52(Rep. Num17), KP54(Rep.Num18), KP55(Rep.Num19), KP57(Rep. Num20), KP58(Rep.Num21), KP60(Rep.Num22), KP61(Rep. Num23), KP63(Rep.Num24)..

CONCLUSION

In summary, the results showed a description of the formation of productive connective thinking schemes when these students undergo the phases of mathematical problem-solving in four phases schema formation connective thinking seen in mathematical problem-solving processes.

(i) Phase of Cognition, shows the relationship between the immediate problem and the intention to explore the direction of problem-solving. The structure of students' thinking is in line with the structure of the problem given. Ideas that emerge on the stage of understanding the causes of interrelated problems build new and varied ideas.

(ii) Phase of Inference, find suitable information and basis for the solving and make the inference be reasonable and logical. Ideas are formed based on information, new ideas, solution ideas, generalization ideas, and experience-based ideas. The ideas that have the same relevance and meaning build a strong network of connective thinking.

(iii) Phase of Formulation, verify the problem and acquire the knowledge and schema. The scheme of connective thinking that is formed is a generalization scheme that is equipped with high spatial abilities and abstraction capabilities.

(iv) Phase of Reconstruction, look back, evaluate, and reconstruct the whole process of problem-solving, and create the new problem. Reconstruction of the scheme of connective thought networks has been formed into a new scheme of connective thinking, then the scheme connects thinking in the domain of more complex problem solving

Implications

Students' thought structure is found to bring up ideas when building mathematical connections. Ideas that can fill the construction gaps practically can be used by the teacher as an indicator to develop learning strategies for students who have the same tendency in thinking. The teacher should help their students solves the problems with various ways of thinking from mathematical problem solving as connections. Theoretically, the findings of this study are expected to contribute to the development of the theory of connection thinking in mathematics learning based on problems solving.

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